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The Evolution of Asset Allocation

A Timeline of Portfolio Optimization Solutions

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(A Timeline of Portfolio Optimization Solutions)

By

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With support from

Smart Portfolios, LLC

September 10, 2004

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Abstract

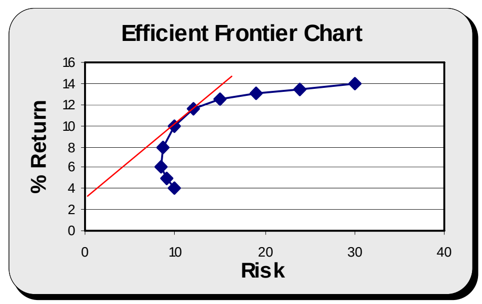
This portfolio optimization research project was conducted to document the evolution of optimization methodologies and to gain an understanding of the commercial market of portfolio optimization solutions. To rank the optimization solutions commercially available we created a proprietary rating system to measure the strengths and weaknesses of each attribute used in the building of an optimization methodology. After ranking these attributes, optimization solutions are measured, scored, and then placed along an evolutionary time continuum. This continuum results in two deliverables: 1) a report of the modern history of asset allocation entitled, "The Evolution of Asset Allocation", and 2) a comparative analysis of optimization solutions.

Introduction

Section 1 of this report illustrates a brief historical perspective on the creation of Modern Portfolio Theory (MPT) and its evolution over time to arrive at current state-of-the-art approaches. Section 2 reviews the process of identifying the commercially available portfolio optimization solutions, and then defines each solution's capabilities relative to the peer group. Section 3 discusses our methodology for quantifying the optimization attributes of each firm. Section 4 describes our time continuum, 'The Evolution of Asset Allocation' and the results of our research.

Historical Perspective

The timeline of portfolio optimization begins with Mean-Variance Optimization (MVO), which was pioneered by Harry Markowitz in 1952. MVO utilizes the expected return and the variance of each asset along with the correlation matrix of the assets in a portfolio. Through the use of MVO, Markowitz produced a return/variance efficient portfolio that suggests a combination of assets with the optimum balance between portfolio risk, measured by the variance or standard deviation, and the return of the portfolio. The graphical representation of the relationship between return and risk is known as the Efficient Frontier. The Efficient Frontier became the foundation of Modern Portfolio Theory (MPT), which Markowitz developed in 1959. The goal of MPT is to reduce risk through diversification while maximizing risk-adjusted returns. The Markowitz MVO model has been the financial community's workhorse in optimizing the Efficient Frontier for over 45 years.



In 1964, the Capital Asset Pricing Model (CAPM) was introduced by William F. Sharpe to describe the relationship between risk and expected return. CAPM states that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat the required return then the investment should not be undertaken.

Valuation with the CAPM uses a variation of discounted cash flows. By varying the discount rate you can alter your projections to compensate for your investment's riskiness. There are different ways to measure risk; the original CAPM defined risk in terms of volatility, as measured by the investment's beta coefficient.

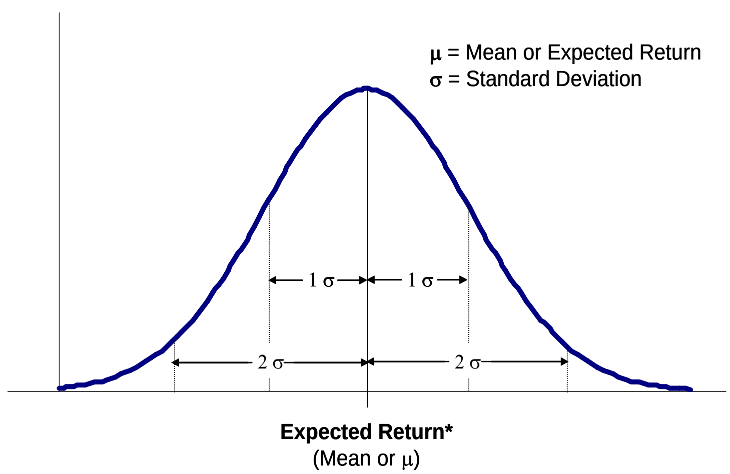
The CAPM formula is:

$$Ra = Rf + beta \times (Rm - Rf)$$

Where: Ra is the risk-adjusted discount rate or Expected Return (or Cost of Capital);
Rf is the rate of a "risk-free" investment, i.e. cash;
Rm is the return rate of a market benchmark, like the S&P 500.

In other words, Ra is the expected return rate you would require before you would be interested in a particular investment at that particular price. The concept is that investors require higher levels of expected returns to compensate them for higher expected risk; the CAPM formula is a simple equation to express that concept.

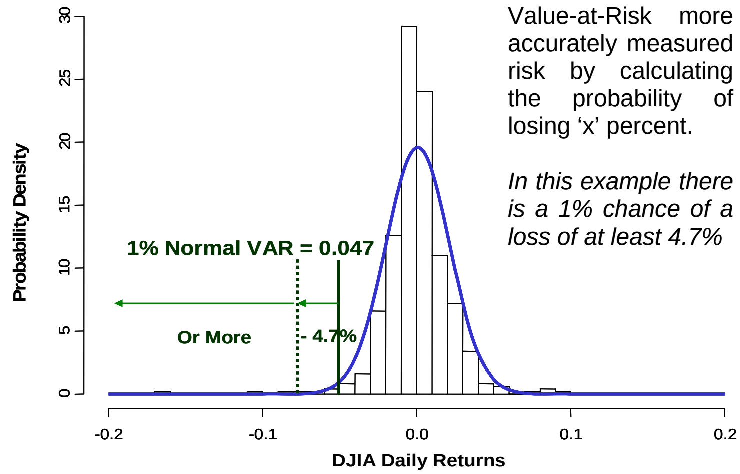
> *"Analysts sometimes use a more complicated value for beta that grows with a company's debt level. There is also lots of controversy about whether beta, which measures past volatility, is sufficient or even relevant in predicting future risk. One big caveat is that the sophistication of the sliding discount rate makes this approach potentially dangerous. If you get creative enough with the discount rate and long-term growth expectations, you can come up with some wildly unrealistic valuations. So if you do use this approach, you should check your results by using discounted cash flows the traditional way, just to make sure you aren't fooling yourself. Also, CAPM is best suited when used in conjunction with MPT and not with valuing individual securities."*
>
> *http://www.moneychimp.com/articles/valuation/capm.htm*



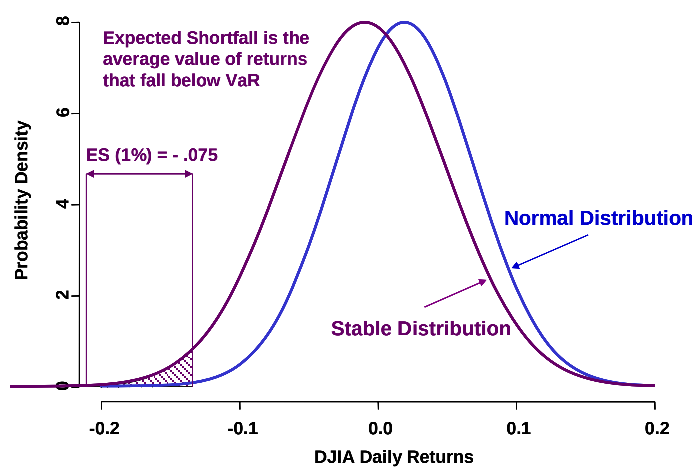
As previously mentioned, the original basis for estimating risk in the model has been through the measure of variance, which is used in mean-variance optimization to generate the Efficient Frontier. The dynamics of MPT has evolved with the industry's more in-depth understanding of risk management.

Arbitrage Pricing Theory (APT) introduced by Roll & Ross in 1982 factors in additional systematic risks such as industrial production, economic news, employment rates, and corporate and individual spending. The concept of APT is to reduce both systemic and non-systemic risk and is an improvement on MPT.

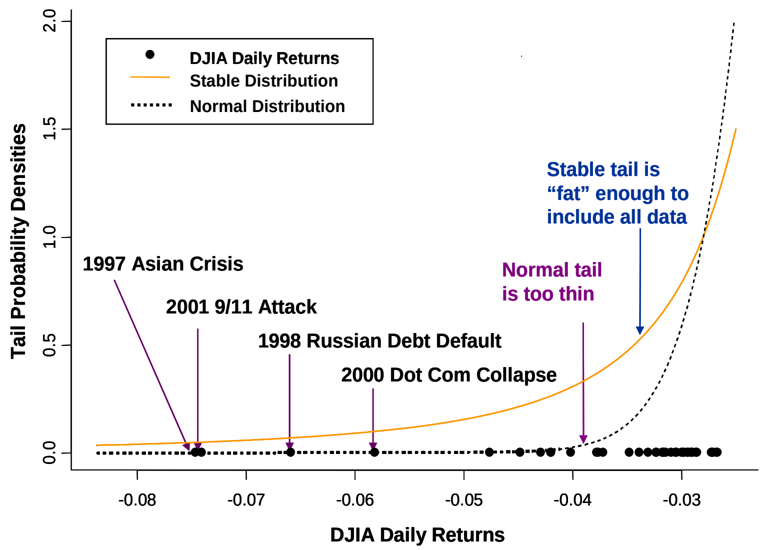
Risk



In 1994, JP Morgan popularized a new way to measure risk called Value at Risk (VaR) as an improved way for institutions to quantify risk in their daily trading activities. VaR describes risk more accurately by means of estimating the minimum loss at a specific probability level. This is more intuitive than variance or standard deviation because the risk is expressed in dollar terms. The main drawback of VaR has been that it only describes a minimum amount of loss and therefore does not specify how big the expected loss could be. However, starting in the late 1990's, the general VaR measure has been complemented with various modified forms such as Historical VaR, Marginal VaR, Conditional VaR, and others; thus improving this form of risk measurement. Recently, an alternative measure has been advanced that goes beyond VaR, called Expected Shortfall (ES). ES is the average value of total losses beyond a define probability level. It not only measures the minimum loss, but the amount of expected loss beyond VaR.

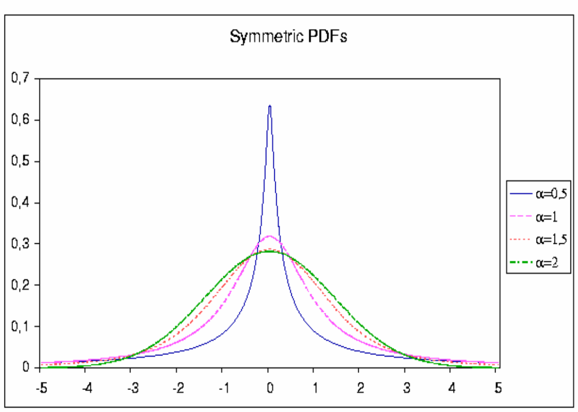


ES is used as a basis for the latest evolution of risk measurement. Risk measurement using ES is one of the major components that convert MPT into the current state-of-the-art methodology, known as Dynamic Portfolio Optimization (DPO), which may furnish more realistic market assumptions and thus providing better investment decisions.

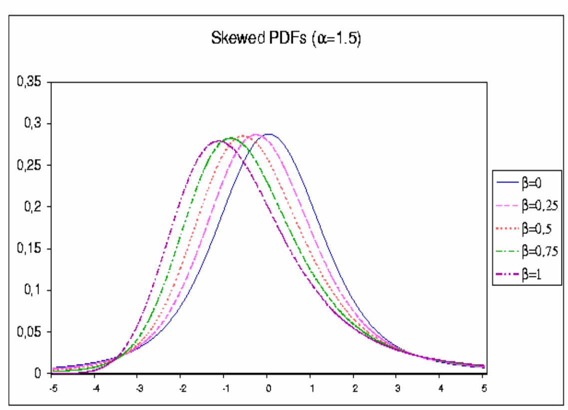


Risk measurement is an important attribute of optimization modeling because of its overall impact on performance (risk vs. return). Estimating risk using standard deviation implies normal distributions with finite variance. Calculating ES or VaR, though, is not restricted to finite variance distributions such as Stable Distributions. A Stable distribution is a distribution which differs in shape from the normal (Gaussian) distribution. In that, Stable distributions can accommodate heavy-tailed (leptokurtic) and skewed data typical for financial time series.

Symmetric Stable Distribution



Skewed Stable Distribution

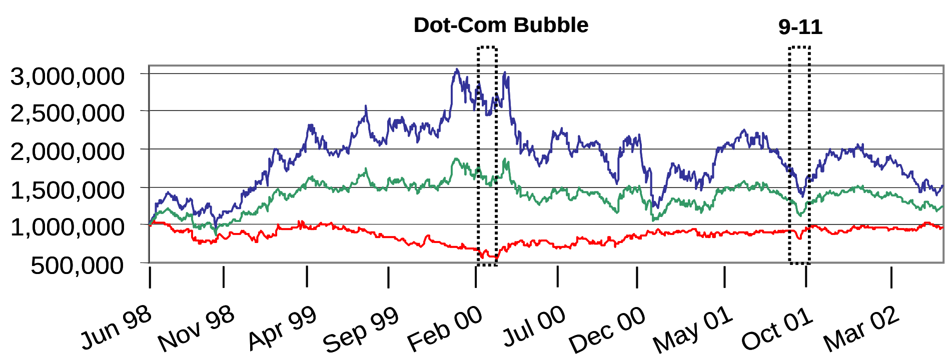


Stable distributions where introduced to finance by Benoit Mandelbrot in1963 and Eugene Fama in 1965 as a superior fit to assets log-returns capable of capturing the heavy-tailed (leptokurtic) behavior of security returns. It was not until 2000 when the financial industry was able to move from the conceptual phase of distribution theory to the actual application through technology.

Diversification

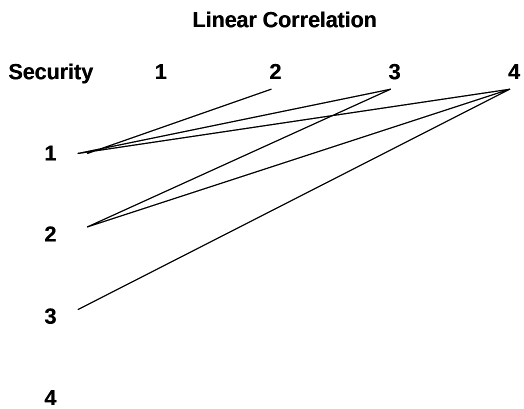
Since asset allocation models rely on diversification as the underlying principal for reducing risk, the attributes that measure diversification need close inspection. The importance of diversification in reducing non-systemic risk was brought to light by Brinson, Beebower & Singer in 1986, 1991, and 1994 was recently reaffirmed by a recent report from Ryan Labs, Inc. in 2003 demonstrating that at least 91.5% of the variation in return is attributed to diversification. The standard attribute used for diversification is correlation. (Commonly, one refers to the linear correlation.)

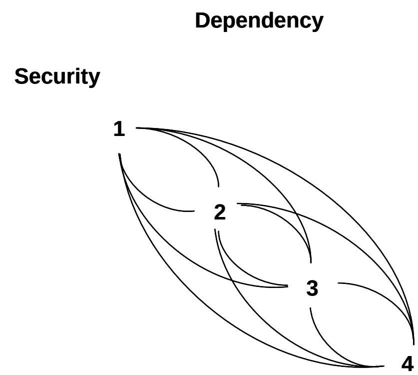
Correlation of 2 Securities over 3 ¾ years



		Return	Standard Dev.	Correlation
MSFT	——	10.9%	46%	
BRK	——	-1.0%	31%	
50/50 Mix	——	5.5%	35%	-.50

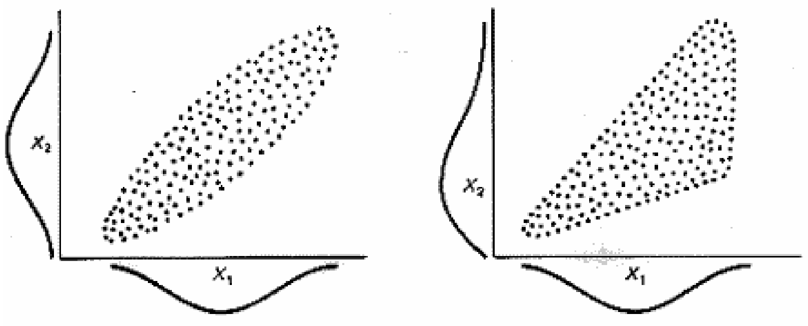
Unfortunately, correlation estimates have been meaningless during periods of extreme events like September 11, 2001, and the Russian Bond default (ala the demise of Long-Term Capital). In both cases, securities that were statistically negatively or non-correlated moved in the same direction upon adverse market conditions. The observed market behavior has required the utilization of a new attribute that would encapsulate dependencies upon extreme market conditions; a solution is found by using Copula functions. Copula Dependencies go beyond linear correlation by capturing the complex inter-relationships between securities.





Distribution Choices and Variance Forecasting

The choice of distribution and the choice of forecasting techniques for describing financial time series are also important. A key parameter of every distribution intimately related to estimating risk, is the scale parameter. (In the case of normal distribution the scale parameter is the standard deviation.) The forecasting of the scale parameter can be achieved by considering Generalized Autoregressive Conditional Heteroscedastic (GARCH) models, or by extracting implied scale parameters (volatilities) from option prices. The common plethora of distributions used include: Normal, Stable, Student-t, and mixtures of distributions. For example, the use of stochastic volatility models within the Black-Scholes approach for option pricing provides an improved fit to market data in terms of implied volatility smile (skew). Even normal distributions with variable scale parameters can accommodate leptokurtic data and can lead to dynamic dependencies.



Homoscedasticity with both variables normally distributed

Heteroscedasticity with skewness on one variable

The following example is intended to help explain the importance of time-weighted attributes when handling historical data. Prior to the 2004 Olympics held in Athens, the United States had never missed the Gold Medal round in the history of Men's Basketball and had only lost two games since professional athletes were allowed to play in 1984. Due to their stellar historical record, a normal distribution would have suggested a very high probability of the team competing in the Gold Medal round, with little risk of losing. The losses the team incurred early in the Athens games would have been construed as statistical outliers, especially since 1984, and would have carried little significance in determining who might win the Olympic Gold Medal. Had GARCH been used instead, the early losses in the Athens Games would have carried more meaning (value) and thus demonstrated that the risk of loss was far greater than the normally distributed historical data would have implied. Therefore, GARCH more accurately defines the current level of risk when combined with Expected Shortfall and is quicker to adapt to more recent events.

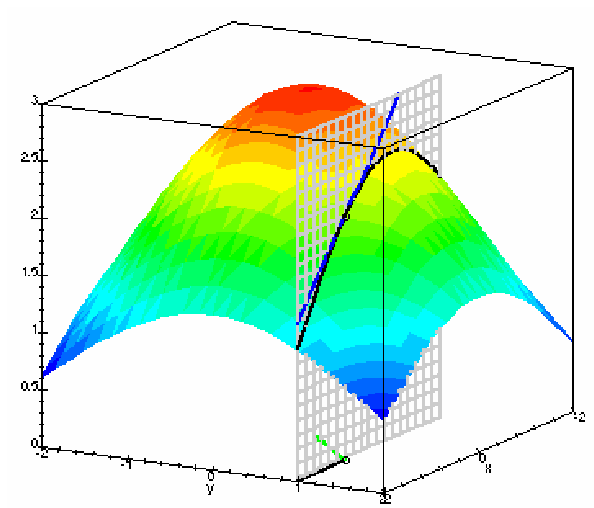
Section Summary

The utilization of these combined attributes are the building blocks for constructing an asset allocation mix whereas the best combination of these attributes, as mentioned above, create the best optimization methodology; that of DPO. In the time between MPT and DPO there have been other methodologies developed to optimize portfolios that have unique approaches to asset allocation, such as Digital Portfolio Theory (DPT) and Mark to Future (MtF) to name two. These theories each make use of unique proprietary processes to optimize the risk-return trade-off and are generally distributed by their founding firms; hence they are not well established in the asset management community.

The mathematical differences between MPT and DPO are evidenced by the volume of lines of code and intense computational processing power required to optimize a portfolio.

Graphically, the latest in asset allocation methodologies is a multi-dimensional representation of the complex attributes that more effectively manages risk and optimizes the efficient frontier.

Mapping the Efficient Frontier



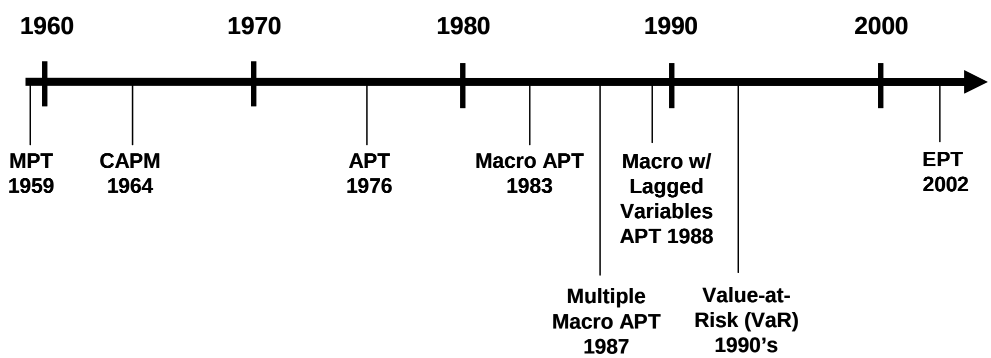
Assessment

In this section we discuss the process for quantifying the attributes of each of the different portfolio optimization solutions. The purpose of optimizing a portfolio is to achieve the maximum return with the minimum amount of risk. With this in mind, we placed heavy weighting on the ability to access clean data and make assumptions about the data, such as Dynamic Parameter Estimation (GARCH) and Leptokurtic Distributions, and the method by which data was described, such as Copula-based Dependence.

A Paradigm Shift in Asset Allocation Models

	Old		New
Distributions:	Normal Distributions	→	Stable Distributions
Data Distribution:	Normal Distribution	→	EWMA/ GARCH
Diversification:	Normal Correlation	→	Copula Dependency
Risk Measurement:	Value-at Risk (Sigma)	→	Expected Shortfall (ES)
Inception:	Markowitz 1959	→	DPO 2003
Theory:	Modern Portfolio Theory	→	Dynamic Portfolio Optimization

The Evolution of Optimization Methodologies



Risk measurement is a required element for building an optimization solution. An optimization solution can only be as good as its risk measurement methodology. For example, a solution may employ some of the latest and most advanced methods in simulation technology, but if the risk measurement calculation is based on antiquated methods, that particular portfolio optimization solution's performance will be inferior to a portfolio optimization solution that utilizes advanced risk measurement methods. This is why risk measurement is the initial attribute we use to classify the different portfolio optimization solutions. This is done by placing a solution into a category based on its risk measurement sophistication. The more advanced the risk methodology the higher the category or "generation". There are a total of four generations to fit the four evolutionary advancements in risk measurement progression. This evolutionary process is repeated for each of the four attributes. Interestingly, we can note how the advancement in each of the four basic attributes coincides with each other; the results are the framework for this report.

In summary, we have divided the optimization solutions into four attributes. Each attribute is then classified by its relative strength or weakness based on whether its methodology is old or new. Any one attribute may have several elements that compose the attribute.

Risk Measurement

The *First Generation* risk measurement methodology is standard deviation, a very straightforward and familiar measure of risk that has been used since the beginning of MPT. Markowitz (1952) used standard deviation in his MVO calculations when first generating the Efficient Frontier.

The *Second Generation* methodology, Value-at-Risk (VaR), replaced standard deviation as a more accurate way of measuring risk by examining the probability of losing money. VaR is calculated assuming a Normal distribution of returns. The inadequacy of the Normal distribution is well recognized by the risk management community and newer forms of VaR have been developed using non-Normal distributions. These newer measures of VaR comprise the *Third Generation* along with other forms of modified VaR such as Historical VaR, Marginal VaR, Conditional VaR, to name a few. These types of modified VaR calculations go beyond the standard measure of VaR with a Normal distribution; any portfolio optimization solution with these capabilities is interpreted to provide a better way to quantify risk and is deserving of a different classification.

The *Fourth Generation* is reserved for portfolio optimization solutions that are able to measure risk more precisely than VaR without making any assumptions of normality of distributions. Expected Shortfall (ES) and Expected Tail Loss (ETL) are measures of risk that fall in this generation; they are the average value of returns that fall below VaR. They not only measure the probability of loss, but the amount of expected loss, unlike VaR, which only provides a minimum amount.

These generations represent an evolution of how risk measurement has progressed over time, with the latest and most advanced methods represented in the fourth generation. Within these generations we looked at the three other attributes identified previously, in no particular order, starting with diversification capabilities.

Diversification

The choice of diversification methodologies determines how the portfolio optimization solution establishes dependency between assets in the portfolio. The basic methodology employed by Markowitz in creating the Efficient Frontier was a covariance matrix that was used to measure the correlation between assets. This is the basic methodology employed in MPT and is based on a linear relationship between assets. Some portfolio optimization solutions have advanced this concept by customizing the correlation measurement in the form of local correlation or weighted average correlation to form a stronger indication of dependence between assets and are viewed as superior to covariance matrix originally employed by Markowitz.

The next progression was the use of genetic algorithms for optimization, which require more computational capabilities, but created an even better allocation. The latest advance in portfolio optimization has been the use of copula dependencies, which are a non-linear measure of dependence and do not require multivariate normal distributions as correlation does.

Copulas are a better measure of dependency because of the well-known fact that in times of extreme market moves assets tend to increase in correlation by moving in the same direction. The impact to an asset or portfolio during extreme events is completely ignored under traditional correlation models making normal correlation less effective as a measurement of risk. The use of copula dependencies allow for a more accurate measure of risk and return and are viewed as the best form of diversification methodology.

Data Distribution

How the portfolio optimization solutions handled data distribution was considered next. At the low end of the spectrum is a Normal distribution followed by single-period MVO. In the middle is the capability to use multi-period MVO, whereby a portfolio is rebalanced to a specified allocation at the end of each period. At the high end of the spectrum are portfolio optimization solutions that can handle multiple horizon models, exponentially weighted moving averages (EWMA), or more sophisticated methodologies in the realm of GARCH. Our research shows a trend towards finding a dynamic model that can find the optimum time sequence relative to a specified data sample and its volatility characteristics based on the overall optimization objectives; obviously a day-trader needs differ from a fund-of-funds manager. Determining the optimal time horizon, also known as Time Parameter Estimations or the Scale Parameter, is perhaps the most daunting task in portfolio optimization.

Variance Forecasting (Simulation Capabilities)

A Monte Carlo Simulation is required once you stray from a finite variance model, such as normal distribution and most VaR models. This is because the number of possible results using a stable distribution is considered infinite. A run of 10,000 to 20,000 simulations are common in advanced optimization models.

The predominant form of simulation we encountered was Monte Carlo Simulation (MCS). MCS replaces Markowitz's Mean Reversion formula on data distributions for portfolio optimization. Monte Carlo simulations were incorporated into asset allocation models to predict future results by simulating thousands of possible portfolio values. In other words, MCS is basically a series of statistical simulations on random samplings of 'histories' to determine the probability of specific outcomes. The numbers of simulations run in a given application ranged from basic, with 10 assets and 100 variables, to advanced, with unlimited assets and variables. Other forms of simulation we encountered involved the employment of historical data, generalized autoregressive conditional heteroskedastic (GARCH) models, and autoregressive conditional heteroskedastic (ARCH) models. It is important to note that a fair number of providers offered no simulation model at all. Portfolio optimization solutions with no simulation capabilities were viewed as inferior to those with simulation within each classification and we viewed those with GARCH/ARCH models as being the most sophisticated along with MCS that had unlimited constraint capabilities.

Identify & Quantify

To begin the study we needed to know who the providers are. The first step was to try and identify all the providers of commercial portfolio optimization solutions. We accomplished this by searching the Internet with various key word combinations (portfolio optimization, etc.) and visiting all websites relating to portfolio optimization to see if a product was offered. We also visited Internet chat sites to engage academics and professionals in computational finance to try and uncover additional firms. We created an extensive list of all of the providers (50 firms) we could identify and the portfolio optimization solutions they offered.

As noted in the previous section there are four basic elements (attributes) required for structuring an asset allocation model: Risk measurement, diversification methodology, distribution of data, and variance forecasting. Having worked with various portfolio optimization solutions in the past, we drew upon this experience to help guide us in deciding what attributes distinguish one portfolio optimization solution from another. Distribution of data and optimization efficiency were similar enough that we decided to combine them into one element, data characterization, when appraising the different portfolio optimization solutions. We wanted to identify the following information about each solution: 1) Risk measurement approach, 2) diversification methodology, 3) data characterization, and 4) forecasting methodology. We included variance forecasting to give us an indication of the viability of the solution and in some instances, the sophistication of the technology (It appeared that the cheaper portfolio optimization solutions were less sophisticated). These are the four main attributes we believe are the underlying foundation of most portfolio optimization solutions, especially the portfolio optimization solutions that are based on MPT or its successor theory.

The next step was how to determine what attributes each of the portfolio optimization solutions utilized. We created a questionnaire as a guide in assessing each portfolio optimization solution. In addition to the information mentioned previously, sections of the questionnaire were devoted to the usage of the portfolio optimization solution, such as: what additional tools, functionality and support were available. We wanted to distinguish if the portfolio optimization solution was a PC based software module or some type of web hosted ASP. This generally had an effect on the pricing of the solution; the PC based solutions typically have a one-time license fee while the web hosted solutions usually required some form of recurring license (annually, quarterly, or monthly). We were also interested in learning if market data was provided and if so, what was the quality (accuracy) of the data and was the solution available separate from the data feed. Once again, this generally had a great effect on the pricing, with data being a major contributor to the price. Support services were not a required element, but specialized technology solutions can be quite complicated and we were interested to see the level of assistance provided. Quite a few of the portfolio optimization solutions were customizable and those that were tended to provide more support services. Several of the more sophisticated technology firms reported that the fastest growing segment of their firms was outside the scope of selling product and more towards a service provider model. The services provided include consulting and customization.

Next we visited each of the solution provider's websites and filled out the questionnaire as best as possible with the information available. All providers on the list were contacted by phone or email to fill in information not readily available from the websites. In some instances the questionnaire was emailed directly to the providers. In other instances we had to email the providers our questions by means of feedback forms and information requests provided by the website because no email address was listed. Because a lot of the portfolio optimization solution providers are small firms it was not uncommon to corresponded or speak directly with the person (typically a PhD) that developed the portfolio optimization solution. Some of the firms we spoke with involved conversations with sales people, but we were usually then transferred to technical support personnel or developers who were able to provide the detailed information we were seeking. Not all firms we identified were contacted; this is because many of the providers had painstakingly simple solutions that did not warrant further due diligence. In addition, there were a few providers that did not respond to our queries. In these instances we did our best to gather the information requested in the questionnaire from the information available on the websites.

The Evolution of Asset Allocation Attributes

We did not set out to create a timeline of portfolio optimization solutions, but once we began classifying solutions by how risk was measured, we realized that this was exactly what we had. We then realized that each classification represented a different generation in the evolution of asset allocation. The classifications were relabeled in terms of when the risk measurement metric came into use:

1^{st} Generation:	*Standard Deviation*
2^{nd} Generation:	*Value at Risk*
3^{rd} Generation:	*Modified Value at Risk*
Next Generation:	*Expected Shortfall*

These generations then form a timeline; evolving from left to right beginning with the First Generation, ending with the latest and most sophisticated risk measuring solutions in the Next Generation. This process was repeated for each of the four attributes. This then forms the basis for our Evolution of Asset Allocation; all we need now is to determine which generation each portfolio optimization solution belongs in.

To start the process we issued a maximum value to each attribute based upon the importance of the attribute to the ideal optimization methodology. In other words, some attributes were weighed heavier than others due to their importance in creating a better asset allocation mix to reduce risk and increase relative returns, thus their impact to the overall score was greater. These scores coincided with the evolutionary progression of optimization methodologies.

With this process for quantifying the attributes of a portfolio optimization solution complete, each solution was reviewed with the information gathered from the websites, survey, and interviews, and placed in one of the generations based on how the solution was measured and scored. Within each generation each solution was again analyzed to confirm its positioning within its group. The sophistication of the review again used the information gathered from the websites, survey, and interviews. To illustrate the position of a portfolio optimization solution within a generation, the most sophisticated solutions within each generation appear farthest right along the time continuum. For example, a solution with the best simulation technology, data distribution, and diversification methodology, but using standard deviation as a measure of risk would still be placed in the First Generation. However, it would be at the right side of the first generation, representing that it is the most sophisticated portfolio optimization solution that uses standard deviation as a measure of risk, but it would not be considered Second Generation.

Through our search we were able to identify fifty different providers of portfolio optimization solutions. After quantifying the attributes of each portfolio optimization solution the break down between generations is as follows:

Generation	% Firms per Solution	Risk Measurement
1st Generation	50%	Standard Deviation
2nd Generation	20%	Value at Risk
3rd Generation	20%	Modified Value at Risk
Next Generation	10%	Expected Shortfall

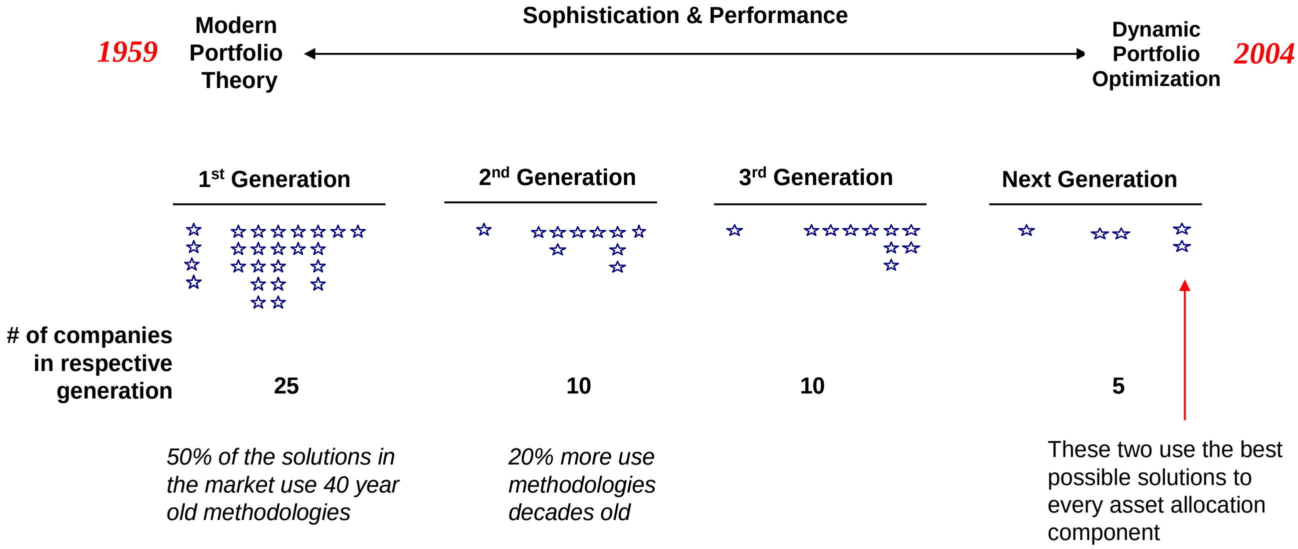
As you can see, half of the portfolio optimization solution providers we could identify are utilizing risk measurement techniques that are over forty years old. Only about one third of the providers are beyond the Second Generation, meaning that only a small number of firms are utilizing the sophisticated risk measurement techniques that have been developed since the mid to late 1990's. Another interesting result of our research is that most of the firms beyond the second generation are small boutique firms that are not well known to the investing public and would take a fair amount of due diligence to find. Most of the well-known larger firm's solutions we could identify fell into the First Generation.

It seems that *caveat emptor* holds true when selecting a portfolio optimization solution, you may be under-optimizing your portfolio and taking on more risk than you realize by not doing the due diligence to find these small boutique providers. As an example, in April of 2004, Morgan Stanley paid $816.4 million to acquire Barra, Inc. Was this acquisition for the purpose of acquiring revenue or for window dressing for Wall Street after the wake of the recent mutual fund scandals? Based on the valuation you may claim it's the later reason, but as a technology solution it is arguably not best of breed.

To our knowledge this is the first paper publicly written on the evolution of asset allocation models or due diligence on optimization solutions. We understand the topic is very complex and has substantial room for error, especially as it relates to proprietary optimization solutions and programs that allow its user to modify potential outcomes through the use of MCS, APT, and the more advanced features offered by the more sophisticated optimization solutions. We have not attempted to provide due diligence on the merits of any one solution provider, but rather attempt to describe what the potential optimization are capable of doing using the attributes claimed by the provider.

Study Results

The Ranking of Portfolio Optimization Solutions



The Evolution of Asset Allocation

Timeline:	**1952 Markowitz Methodology**					**TIME>>>**			**Dynamic Portfolio Optimization**		
Theory:	MPT	CAPM	MCS	APT	Macro APT		DPT	Macro/LV-APT		MtF	PA & EVT
Risk Metric:	Std. Dev.			VaR		Historical VaR		Stable VaR	Con'd VaR		ES
Diversification:	Correlation				GA			MHM			Copula
Data Dist:	Normal		SP-MVO	MP-MVO			EWMA				
Variance 4cast:	Mean Revision (Variance Forecasting)			RGM				RE	Digital		GARCH
Risk Ratios:			Sharpe	Treynor		Sortino					Smart

List of Abbreviations

Portfolio Theories

MPT	Modern Portfolio Theory
CAPM	Capital Asset Pricing Model
MCS	Monte Carlo Simulation
APT	Arbitrage Pricing Theory
>	Macro APT
>	Macro w/ Lagged Variables APT
DPT	Digital Portfolio Theory
MtF	Mark-to-Future
PA	Phi-Alpha
EVT	Extreme Value Theory

Risk Measurement

Std Dev	Standard Deviation
VaR	Value-at-Risk
>	Normal VaR
>	Stable VaR
>	Historical VaR
CVaR	Conditional VaR
ES	Expected Shortfall

Diversification Methodology

Co	Correlation
GA	Genetic Algorithms
Copula	Copula Dependency

Distribution of Data

Normal	Normal Distribution
MVO	Mean Variance Optimization
>	Simple period MVO
>	Multi-period MVO
EWMA	Exp. Weighted Moving Average
MHM	Multiple-Horizon Model

Variance Forecasting

MR	Mean Reversion
RGM	Rebalanced Geometric Mean
RE	Re-sampled Efficiency
Digital	Digital Signal Optimization

The Evolution of Portfolio Optimization Solutions

In the Beginning:

Ra = Rf + beta x (Rm - Rf)

Where

Ra is the risk-adjusted discount rate or Expected Return (or Cost of Capital);
Rf is the rate of a "risk-free" investment, i.e. cash;
Rm is the return rate of a market benchmark, like the S&P 500.

Attributes comprising today's State-of-Art solutions:

1. Univariate Stable Distributes

A stable distribution for a random risk factor X is defined by its characteristic function:

$$F(t) = E\left(e^{itX}\right) = \int e^{itx} f_{\mu,\sigma}(x)dx\,,$$

where

$$f_{\mu,\sigma}(x) = \frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right)$$

Is any probability density function in a location-scale family for X:

$$\log F(t) = \begin{cases} -\sigma^{\alpha}\,|t|^{\alpha}\left(1 - i\beta\,\mathrm{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right) + i\mu t, & \alpha \neq 1 \\ -\sigma|t|\left(1 - i\beta\frac{2}{\pi}\mathrm{sgn}(t)\log|t|\right) + i\mu t, & \alpha = 1 \end{cases}$$

A stable distribution is therefore determined by the four key parameters:

1. α determines density's kurtosis with $0 < \alpha \leq 2$ (e.g. tail weight)
2. β determines density's skewness with $-1 \leq \beta \leq 1$
3. σ is a scale parameter (in the Gaussian case, α =2 and $2\sigma^2$ is the variance)
4. μ is a location parameter (μ is the mean if 1 <$\alpha \leq 2$)

The case α=1 and β=0 yields the Cauchy distribution with much fatter tails than the Gaussian, and is given by:

$$f_{\mu,\sigma}(x) = \frac{1}{\pi\cdot\sigma}\left(1 + \left(\frac{x-\mu}{\sigma}\right)^2\right)^{-1}$$

2. Discrete Time Volatilities based on GARCH Models

A continuous time GARCH model:

$$\beta \int_0^n \exp\left(\sum_{j=\lfloor s \rfloor+1}^{n-1} \log(\delta + \lambda \varepsilon_j^2) \right) ds,$$

where

For each $t \geq 0$ the characteristic function of L_t can be written in the form

$$E(e^{i\theta L_t}) = \exp\left(t \left(i\gamma_L \theta - \tau_L^2 \frac{\theta^2}{2} + \int_{(-\infty,\infty)} \left(e^{i\theta x} - 1 - i\theta x 1_{\{|x| \leq 1\}} \right) \Pi_L(dx) \right) \right), \quad \theta \in \mathbb{R},$$

3. Copula Multivariate Dependence Models

A copula may be defined as a multivariate cumulative distribution function with uniform marginal distributions:

$$C(u_1, u_2, \cdots, u_n), \qquad u_i \in [0,1] \text{ for } i = 1, 2, \cdots, n$$

where

$$C(u_i) = u_i \quad \text{for } i = 1, 2, \cdots, n.$$

It is known that for any multivariate cumulative distribution function:

$$F(x_1, x_2, \cdots, x_n) = P(X_1 \leq x_1, X_2 \leq x_2, \cdots X_n \leq x_n)$$

there exists a copula C such that

$$F(x_1, x_2, \cdots, x_n) = C(F_1(x_1), F_2(x_2), \cdots, F_n(x_n))$$

where

the $F_i\left(x_i\right)$ are the marginal distributions of $F(x_1, x_2, \cdots, x_n)$, and conversely for any copula C the right-hand-side of the above equation defines a multivariate distribution function $F(x_1, x_2, \cdots, x_n)$

Disclaimer

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